INTERNATIONAL WORKSHOP ON

DIFFERENTIAL AND DIFFERENCE EQUATIONS:
Theory, Numerics and Applications

Hanoi and Halong, *29-31 October, 2009*

Main sponsors: Asia Reseach Center, VNU-Hanoi, and Hanoi University of Science

SCIENTIFIC PROGRAM & ABSTRACTS

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DIFFERENTIAL AND DIFFERENCE EQUATIONS: Theory, Numerics and Applications

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Final Announcement

Main sponsors: Asia Reseach Center, VNU-Hanoi, and Hanoi University of Science

Workshop venue: Hanoi University of Science Campus and Ha Long City

Topics:

- *Ordinary differential and differential-algebraic equations*
- *Difference equations*
- *Partial differential equations*
- *Stochastic differential equations*
- *Dynamical systems*
- *Numerical methods and simulations*
- *Modelling, scientific computation and applications, etc*

Scientific program committee: Nguyen Huu Du (Chair), Vu Hoang Linh, Pham Ky Anh, Nguyen Huu Cong, Hoang Quoc Toan, Nguyen Dinh Cong, Dinh Nho Hao

Local organizing committee: Vu Hoang Linh (Chair), Le Cong Loi, Nguyen Thi Hong Minh, Le Huy Chuan, Vu Cong Bang, Nguyen Trong Hieu, Vu Tien Dung

Invited speakers (confirmed): ***Keonhee Lee*** (Chungnam University, Korea), ***Manseob Lee*** (Mokwon University in Daejeon, Korea), ***Noboru Okazawa*** (Tokyo University of Science, Japan), ***Louis Chen*** (National University of Singapore, Singapore), ***Duong Minh Duc*** (Vietnam National University, Ho Chi Minh City), ***Nguyen Khoa Son*** (Institute of Mathematics, Hanoi), ***Mai Duc Thanh*** (Vietnam National University, Ho Chi Minh City), ***Nguyen Thieu Huy*** (Hanoi University of Technology), ***Dang Duc Trong*** (Vietnam National University, Ho Chi Minh City), ***Hoang Quoc Toan*** (Vietnam National University, Hanoi), ***Luu Hoang Duc*** (Institute of Mathematics, Hanoi), ***Nguyen Manh Hung*** (Hanoi University of Education, Hanoi)

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INTERNATIONAL WORKSHOP ON

DIFFERENTIAL AND DIFFERENCE EQUATIONS: Theory, Numerics and Applications

29-31 October, 2009

SCIENTIFIC PROGRAM

1rst Day (Thu, 29.10), HUS Campus - Hanoi:

Meeting Room 418, Building T1, 334, Nguyen Trai, Thanh Xuan, Hanoi

8.00-8.30:	Registration
8.30-8.45:	Opening Ceremony
8.45-11.45:	**Selected lectures on Dynamical Systems**
	Chair: Nguyen Huu Du
8.45-9.25:	***Keonhee Lee*** (Chungnam University, Korea): *Global Dynamics Beyond Uniform Hyperbolicity*
9.25-10.05:	***Manseob Lee*** (Mokwon University in Daejeon, Korea): *Diffeomorphisms with C^1-stably average shadowing*
10.05-10.15:	Coffee break
	After break, Chair: Keonhee Lee
10.15-10.45:	***Luu Hoang Duc*** (Institute of Mathematics, Hanoi): *Hyperbolicity and invariant manifolds for planar nonautonomous systems on finite time intervals*
10.45-11.15:	***Khosro Tajbakhsh*** (Chungnam University, Korea): *C^1-Generic Diffeomorphisms with Specification Property*
11.15-11.45:	***Le Huy Tien*** (Chungnam University, Korea): *Hyperbolicity of Chain Components and Homoclinic Classes for C^1-Vector Fields*
12.00:	Travel by bus to Ha Long City (lunch will be organized during the trip)

INTERNATIONAL WORKSHOP ON

DIFFERENTIAL AND DIFFERENCE EQUATIONS: Theory, Numerics and Applications

29-31 October, 2009

2nd Day (Fri, 30.10), Halong City: Conference Hall, Công Đoàn Hotel

8.30-12.00:	**Morning Session, Chair: Pham Ky Anh**
8.30-9.10:	***Nguyen Khoa Son*** (Institute of Mathematics, Hanoi): *Controllability radius of linear systems under structured perturbations*
9.10-9.50:	***Noboru Okazawa*** (Tokyo University of Science, Japan): *Linear evolution equations of hyperbolic type in Hilbert space with applications to symmetric hyperbolic systems*
9.50-10.30:	***Nguyen Thieu Huy*** (Hanoi University of Technology): *Exponentially dichotomous generators of evolution bisemigroups and perturbations*
10.30-10.40:	Coffee break
	Chair: Nguyen Van Mau
10.40-11.20:	***Duong Minh Duc*** (Vietnam National University, Ho Chi Minh City): *Global eigenvalue-crossing and multiplicity of solutions to elliptic equations*
11.20-12.00:	***Dang Duc Trong*** (Vietnam National University, Ho Chi Minh City): *Some results on backward heat equations*
12.00-14.00:	Lunch break
14.00-17.00:	**Afternoon Session, Chair: Duong Minh Duc**
14.00-14.40:	***Hoang Quoc Toan*** (Vietnam National University, Hanoi): *On some semilinear elliptic problems with singular potentials involving symmetry*
14.40-15.20:	***Mai Duc Thanh*** (Vietnam National University, Ho Chi Minh City): *Admissible shock waves and traveling waves of conservation laws with diffusion and dispersion coefficients*
15.20-15.30:	Coffee break

Chair: Nguyen Huu Cong

15.30-16.10: ***Louis Chen*** (National University of Singapore, Singapore): *From Stein identities to moderate deviations*

16.10-16.50: ***Nguyen Manh Hung*** (Hanoi University of Education, Hanoi): *Cauchy-Neumann problem for the hyperbolic systems in cylinders with base containing conical points*

16.50-17.20: ***Shangwen Lin*** (Tamkang University, Taiwan): *Positive Solutions of Three-Point Boundary Value Problems for Third-Order DifferentialEquations on Time Scales*

19.00: Workshop dinner

3rd Day (Sat, 31.10), Halong Bay:

09.00-15.00: Visit Ha Long Bay by boat (lunch on boat)

15.00-18.00: Travel back to Hanoi by bus

INTERNATIONAL WORKSHOP ON

DIFFERENTIAL AND DIFFERENCE EQUATIONS:
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ABSTRACTS

Global eigenvalue-crossing and multiplicity of solutions to elliptic equations

Duong Minh Duc - Department of Mathematics and Computer Science, University of Science , Hochiminh City (Vietnam)
Nguyen Hoang Loc - Department of Mathematics, University of Utah (USA)
Nguyen Le Luc - Mathematical Institute, Oxford University (UK)
Le Quang Nam - Department of Mathematics , Columbia University, New York (USA)
Truong Trung Tuyen - Department of Mathematics, Indiana University (USA)

Abstract.

We introduce the concept of global eigenvalue-crossing and use it to study the problem on multiple solutions of asymptotically linear elliptic equations $\Delta u + f(x,u) = 0$ involving $(S)_+$ operators. Our method can be applied to the case that $\frac{f(x,u)}{u}$ globally but not pointwise crosses any λ_i for any x in a part of Ω when u varies from $-\infty$ to ∞, that is, $\frac{f(x,u)}{u}$ may not cross any λ_i for every x in this part and $\lim_{u\to\infty}\frac{f(x,u)}{u}$ may be equal to some λ_j for every x in another part of Ω.

Key words : Index of critical points. Mountain-pass type. Nonlinear elliptic equations. Multiplicity of solutions. Kato's class.

AMS Subject classification : 47H11, 55M25, 35J20.

On some semilinear elliptic problems with singular potentials involving symmetry

H.Q. Toan and N.T. Chung

Abstract. Using varational techniques, we deal with the existence and multiplicity of solutions for semilinear elliptic problems of the form

$$\begin{cases} -\Delta u &= \frac{\mu}{|x|^2}u + f(x,u) \quad \text{in} \quad \Omega, \\ u &= 0 \qquad\qquad\qquad\quad \text{on} \quad \partial\Omega, \end{cases}$$

where $\Omega = \Omega_1 \times \Omega_2 \subset \mathbb{R}^N$ ($N \geqq 5$) is a bounded domain having cylindrical symmetry, $\Omega_1 \subset \mathbb{R}^m$ is a bounded regular domain and Ω_2 is a $k-$dimensional ball of radius R, centered in the origin and $m + k = N$, and $m \geqq 2$, $k \geqq 3$, $0 \leqq \mu < \mu^\star = \left(\frac{N-2}{2}\right)^2$.

Global Dynamics Beyond Uniform Hyperbolicity

Keonhee Lee

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Uniformly hyperbolic systems are nowadays fairly well understood, both from the topological and the ergodic point of view. Outside the hyperbolic domain, two main phenomena occur: homoclinic tangencies and cycles involving saddles with different indices. Homoclinic classes and chain compoments are the natural candidates to replace hyperbolic basic sets in non-hyperbolic theory. Several recent papers explore their "hyperbolic-like" properties, many of which hold only for generic dynamical systems. In this talk, we study how a C^1-robust dynamic property (i.e. a property that holds for a system and all C^1 nearby ones) on the underlying manifold would influence the behavior of the tangent map on the tangent bundle.

References

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[3] K. Lee and M. Lee, *Hyperbolicity of C^1 stably expansive homoclinic classes*, to appear in Discrete & Continuous Dynam. Sys.

[4] K. Lee, K. Moriyasu and K. Sakai, *C^1-stable shadowing diffeomorphisms*, Discrete & Continuous Dynam. Sys. 22 (2008), 683-697.

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[7] K. Lee and K. Tajbakhsh, *C^1-generic diffeomorphisms with specification property*, preprint.

[6] K. Lee and X. Wen, *Shadowable chain transitive sets of C^1-generic diffeomorphisms*, preprint.

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[1]

C^1-Generic Diffeomorphisms with Specification Property

Khosro Tajbakhsh

Department of Mathematics, Chungnam National University, Daejeon, 305-764, Korea

Abstract. Let f be a diffeomorphism of a compact C^∞ manifold. In this paper we introduce the notion of specification property for a closed f-invariant set, and prove that C^1-generically, if f has the specification property on a locally maximal f-invariant set Λ then Λ is hyperbolic. As a corollary, we get C^1-generically, f is Anosov if and only if f has the specification property. Moreover, we show that C^1-generically, f has the specification property on the chain recurrent set $\mathcal{R}(f)$ if and only if $\mathcal{R}(f) = M$ is hyperbolic for f. (This is a joint work with K. Lee)

References

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[1]

Hyperbolicity of Chain Components and Homoclinic Classes for C^1-Vector Fields

Tien Le Huy
Department of Mathematics, Chungnam National University,
Daejeon, 305-764, Korea

In this talk, we introduce the notions of robustly expansive homoclinic classes and shadowable chain components for a C^1-vector field on a compact C^∞ manifold, and study their hyperbolic structures using an extended version of Mane's techniques in [7]. The homoclinic class of a hyperbolic periodic orbit is the closure of the transverse intersections of its invariant manifolds.

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[1] C. Bonatti, N. Gourmelon and T. Vivier, *Perturbations of the derivative along periodic orbits*, Ergod. Th. & Dynam. Sys. 26 (2006), 1307-1337.

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[1]2000 *Mathematical Subject Classification*: 37B20, 37C50, 37D30

From Stein identities to moderate deviations

Louis H. Y. Chen
National University of Singapore

Abstract

Unlike the classical method for probability approximations, Stein's method does not rely on Fourier analysis but on the solution of a functional equation which may be a differential equation, a difference equation or an integral equation. Central to Stein's method is the construction of Stein identities. In this talk I will begin with a discussion on Stein's method and ways of constructing Stein identities, with illustrations from a number of examples including the zero-biased coupling, the binary expansion of a random integer, the anti-voter model, and the Curie-Weiss model. I will then present a Cramer-type moderate deviation result based on a fairly general Stein identity and apply the result to the four examples mentioned above. This talk is based on a joint work with Xiao Fang and Qi-Man Shao.

Title: Hyperbolicity and invariant manifolds for planar nonautonomous systems

on finite time intervals

Luu Hoang Duc

Ha Noi Institute of Mathematics, 18 Hoang Quoc Viet street, Ha Noi

Stefan Siegmund

Institute of Analysis, Department of Mathematics, TU Dresden, 01062 Dresden, Germany.

(published in International Journal of Bifurcation and Chaos, Volume 18, Issue 3 (2008), pp. 641-674)

Abstract: The method of invariant manifolds was originally developed for hyperbolic rest points of autonomous equations. It was then extended from fixed points to arbitrary solutions and from autonomous equations to nonautonomous dynamical systems by either the Lyapunov–Perron approach or Hadamard's graph transformation.

We go one step further and study meaningful notions of hyperbolicity and stable and unstable manifolds for equations which are defined or known only for a finite time, together with matching notions of attraction and repulsion. As a consequence, hyperbolicity and invariant manifolds will describe the dynamics on the finite time interval.
We prove an analog of the Theorem of Linearized Asymptotic Stability on finite time intervals, generalize the Okubo–Weiss criterion from fluid dynamics and prove a theorem on the location of periodic orbits. Several examples are treated, including a double gyre flow and symmetric vortex merger.

Title: **Admissible shock waves and traveling waves of conservation laws with diffusion and dispersion coefficients**

Abstract. Shock waves of conservation laws are admissible under certain admissibility criteria. One of the common admissibility conditions is to require the shock to be the limit of the so-called traveling waves when viscosity and capillarity effects are taken into accounts. This talk presents the method of estimating attraction domain for the existence of traveling waves of conservation laws with diffusion and dispersion, which are represented by viscosity and capillarity coefficients. The left-hand and right-hand states of a shock wave correspond to a stable node and a saddle point of an ordinary nonlinear system of differential equations, supplemented by boundary conditions. The goal is to find a trajectory leaving the saddle point at $-\infty$ and converging to the stable node at $+\infty$, or vice-versa. The method is based on LaSalle's invariance principle. Level sets of a Lyapunov-type function provide us with reasonable estimates of the domain of attraction of the stable node. Then, we show that exactly one stable trajectory of the saddle point enters the domain of attraction of the stable node. This establishes a sole saddle-to-stable connection and therefore gives us a unique traveling wave.

This talk is based on my following recent works:

1. M.D.Thanh, Global existence of traveling waves for general flux function, Nonlinear Anal: TMA, (accepted for publication)
2. M.D. Thanh, Traveling waves of general scalar conservation laws with singular diffusion and nonlinear dispersion, Preprint
3. M.D. Thanh, Global existence of traveling waves of viscous-capillary p-system associate with Lax shocks, Preprint
4. M.D. Thanh, Attractor and traveling waves of a fluid with nonlinear diffusion and dispersion, submitted to Nonlinear Anal.: TMA

[1]

Diffeomorphisms with C^1-stably average shadowing

Manseob Lee
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Daejeon, 302-729, Korea

Recently C. Bonatti, N. Gourmelon and T. Vivier [**?**] proved that a periodic orbit of large period of a diffeomorphism on a compact smooth manifold either admits a l-dominated splitting or can be turned into a sink or a source by a C^1-small perturbation along the orbit. In this paper we show that if f has the C^1-stably average shadowing property on a closed f-invariant transitive set Λ, then Λ admit a l-dominated splitting.

References

[1] C. Bonatti, L. J. Díaz and E. Pujals, *A C^1-generic dichotomy for diffeomorphisms: weak forms of hyperbolicity or infinitely many sinks or sources*, Annals of Math. **158** (2003), 187-222.

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[1]2000 *Mathematical Subject Classification*: 37D30, 37C50

CONTROLLABILITY RADIUS OF LINEAR SYSTEMS UNDER STRUCTURED MULTI-PERTURBATIONS

by

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Institute of Mathematics, 18 Hoang Quoc Viet Rd., Hanoi, Vietnam}

and

Do Duc Thuan
Department of Applied Mathematics and Informatics,
University of Technology, 1 Dai Co Viet Str., Hanoi, Vietnam

Abstract

In this paper we develop a unifying approach to the computation of the controllability radius of linear control systems. By using linear multi-valued operators in representing and estimating the system's equations and matrices involved we are able to derive computable formulas of the distance from a controllable linear system to uncontrollability under the assumption that the system's matrices are subjected to structured multi-perturbations and measured by arbitrary operator norms. In the case of spectral norms, the obtained results unify and extend some previous works as well as a recent interesting result in M. Karrow, D. Kressner, On the structured distance to uncontrollability, Systems and Control Letters, 58**(**2009**)** 128-132. Some illustrating examples are given.

Mathematics Subject Classifications: 06B99, 34D99, 47A10, 47A99, 65P99.

EXPONENTIALLY DICHOTOMOUS GENERATORS OF EVOLUTION BISEMIGROUPS AND PERTURBATIONS

NGUYEN THIEU HUY

Abstract. We will talk about the relation of dichotomies of evolutions families and that of related operators. Concretely, to an evolution family $\mathcal{U} = (U(t,s))_{t\geq s\geq 0}$ of bounded operators on a Banach space X and through the integral equation $u(t) = U(t,s)u(s) + \int_s^t U(t,\xi)f(\xi)d\xi$, we associate an operator G_Z acting on Banach spaces of X-valued functions corresponding to admissible Banach function spaces. These spaces contain the L_p spaces ($1 \leq p < \infty$), the Lorentz spaces $L_{p,q}$ and many other function spaces occurring in interpolation theory. We will show that the exponential dichotomy of $\mathcal{U}$ is equivalent to the exponential dichotomy of the operator G_Z generating a bisemigroup $(\mathcal{T}(t))_{t\in\mathbb{R}}$. We also prove that the exponential dichotomy of G_Z is robust under small perturbations by bounded operators. This leads to applications to vector-valued Wiener-Hopf and to Riccati equations. This is a joint work with Professor Nagel from university of Tuebingen.

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Key words and phrases. Evolution family, integral equations, admissible function spaces, exponential dichotomy, exponentially dichotomous operators, perturbations.

Linear evolution equations of hyperbolic type in Hilbert space with applications to symmetric hyperbolic systems

Noboru Okazawa
Science University of Tokyo

DDE 09, Hanoi, 29-31 October, 2009

Abstract

Let $\{A(t); 0 \le t \le T\}$ be a family of closed linear operators in a complex Hilbert space X. This talk is concerned with linear evolution equations of the form

(E) $$du(t)/dt + A(t)u(t) = f(t) \quad \text{on} \quad [0,T].$$

Let S be a selfadjoint operator in X, satisfying $(u, Su) \ge \|u\|^2$ for $u \in D(S)$. Assume, as a simple case, that the following four conditions are satisfied:

(**I**) There is a constant $\alpha \ge 0$ such that

$$|\mathrm{Re}\,(A(t)v, v)| \le \alpha \|v\|^2 \quad \forall\, v \in D(A(t)) \quad \forall\, t \in [0,T];$$

(**II**) $Y := D(S^{1/2}) \subset D(A(t)) \quad \forall\, t \in [0,T]$;

(**III**) There is a constant $\beta \ge \alpha$ such that

$$|\mathrm{Re}\,(A(t)u, Su)| \le \beta \|S^{1/2}u\|^2 \quad \forall\, u \in D(S) \quad \forall\, t \in [0,T];$$

(**IV**) $A(\cdot) \in C_*([0,T]; B(Y,X))$, where the subscript $_*$ is used to refer the strong operator topology in $B(Y,X)$, the space of all bounded linear operators on Y to X.

Then as for the solvability of (E) we have the following

Theorem. Let $f(\cdot) \in C([0,T]; X) \cap L^1(0,T; Y)$. Then for $u_0 \in Y$ there exists a unique solution $u(\cdot) \in C^1([0,T]; X) \cap C([0,T]; Y)$ of (E) with $u(0) = u_0$, given by

$$u(t) := U(t,0)u_0 + \int_0^t U(t,s) f(s)\, ds,$$

where $\{U(t,s); 0 \le s \le t \le T\}$ is the evolution operator for $\{A(t)\}$.

It is late Professor Tosio Kato who applied his abstract theory (1970) to the Cauchy problem for symmetric hyperbolic systems:

$$\frac{\partial u}{\partial t} + \sum_{j=1}^{N} a_j(x,t)\frac{\partial u}{\partial x_j} + b(x,t)u = f(x,t), \quad (x,t) \in \mathbb{R}^N \times [0,T], \qquad u(x,0) = u_0(x).$$

Here $u = {}^t(u_1, \cdots, u_m) \in X := L^2(\mathbb{R}^N)^m$ is an m-vector of unknown functions of (x,t), $a_j(x,t)$ and $b(x,t)$ are $m \times m$ matrix functions, and $a_j(x,t)$ are assumed to be Hermitian. The above-mentioned abstract theorem yields a new approach to the Cauchy problem of these systems which considerably simplifies the original proof.

Cauchy-Neumann problem for the hyperbolic systems in cylinders with base containing conical points

Nguyen Manh Hung, HaNoi National University of Education

Phung Kim Chuc, CanTho University

Abstract. The goal of this paper is to establish the existence, uniqueness and regularity of solutions of Cauchy-Neumann problem for the hyperbolic systems in cylinders with base containing conical points.

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